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A CRITICAL EVALUATION OF THE CONSTRUCTION OF THE KOBE STRATEGY MATRIX: LESSONS LEARNED FROM BIGEYE TUNA IN THE EASTERN PACIFIC OCEAN

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1. INTRODUCTION

Following the recommendations of the first joint meeting of the tuna regional fisheries management organizations (RFMOs), held in Kobe, Japan, in January 2007, there are ongoing efforts to standardize the presentation of stock assessment results for management advice among the world's RFMOs. One of the main recommendations was to present stock assessment results in the form of "four quadrant, red-yellow-green" format, the so-called Kobe plot (Report of the first joint meeting of the tuna RFMOs). This task has already been implemented with some variants by all tuna RFMOs. The next step is to present a "strategy matrix" for managers that provides alternative options for meeting management targets (Report of the second joint meeting of the tuna RFMOs).

Maunder and Aires-da-Silva (2011) provided a critical evaluation of the Kobe plot and the Kobe strategy matrix strategy matrix, and their application to the assessment and management of tuna in the eastern Pacific Ocean (EPO). This paper identifies and critically evaluates a series of alternative approaches which could be used to implement the Kobe strategy matrix (bootstrapping, Monte Carlo methods, Bayesian MCMC analysis and the normal approximation method). An application of the normal approximation method is illustrated with the base case assessment model for bigeye tuna in the EPO. The analysis is only an illustration of the construction of the Kobe strategy matrix and the results should not be used for management advice.

2. THE KOBE STRATEGY MATRIX

As specified in the Report of the second joint meeting of the tuna RFMOs:

> *"the matrix would present the specific management measures that would achieve the intended management target. The probabilities and timeframes to be evaluated would be determined by the Commission. In the case of fisheries managed under TACs, the outputs would be the various*

TACs that would achieve a given result. In the case of fisheries managed by effort limitations, the outputs would be expressed as, for example, fishing effort levels or time/area closures, as specified by the Commission. It would also indicate where there are additional levels of uncertainty associated with data gaps. Managers would then be able to base management decisions upon the level of risk and the timeframe they determine are appropriate for that fishery.”

The Kobe strategy matrix is formatted to provide the management measures that will produce a desired outcome. This differs from traditional decision tables that provide performance measures for a set of alternative management actions under different states of nature (Punt and Hilborn 1997). For example, the Kobe strategy matrix might present the catch levels that have a 95% probability of being above B_{MSY} in 10 years. It also focuses on providing the management measures for different fixed probability levels and for different time frames. In contrast, a traditional decision table might present the probability of being above B_{MSY} in 10 years under catch levels of 1000 t, 1500 t, and 2000 t, for different assumptions about the steepness of the Beverton-Holt stock-recruitment relationship (the states of nature). The Kobe strategy matrix could be repeated for different states of nature to provide information about the sensitivity of the management measures to model assumptions (states of nature).

The Kobe strategy matrix provides the same type of information as traditional decision tables, and if the tables were comprehensive enough, they would provide essentially the same information. The main difference is in how they are calculated. A traditional decision table is much easier to calculate; for example, all that is needed is a set of stochastic projections for each alternative management action. In contrast, creating the Kobe strategy matrix requires stochastic projections under a reasonable range of management actions (*e.g*. catch levels) to determine which management action provides the desired probability. Shortcuts, such as interpolation between values or search algorithms, can be used to reduce the number of management actions that need to be simulated.

3. SOURCES OF UNCERTAINTY

The Kobe strategy matrix explicitly requires the estimation and evaluation of uncertainty. By presenting the probability of meeting a target reference point, for example, the Kobe strategy matrix accounts for uncertainty in evaluating the status of the stock. There are several different sources of uncertainty, but the main ones are parameter estimation uncertainty, model structure uncertainty, and future process variation. Parameter estimation uncertainty and the main source of future processes variation (recruitment variation) are generally well determined in stock assessment models; it is the model structure uncertainty that is often poorly represented. In general, sensitivity analyses to model structure assumptions are used to provide information on the uncertainty about model structure. However, unless probability statements can be assigned to the different model structure assumptions, it is difficult to formally include model structure uncertainty into the Kobe strategy matrix. A separate Kobe strategy matrix could be created for each model structure assumption (state of nature), similar to a standard sensitivity analysis for a stock assessment model. Alternatively, if probability statements can be assigned, the model structure uncertainty could be integrated into a single Kobe strategy matrix.

Since the Kobe strategy matrix is formulated based on probability statements, the estimation of uncertainty is critical in their creation. Typically, uncertainty will be underestimated, particularly if model structure uncertainty is not taken into consideration. Underestimation of uncertainty will drive the Kobe strategy matrix in a certain direction depending on how the management reference points and probability statements are developed. For example, if the probability statement is that there is a 50% probability that the stock is above a biomass target, then underestimating uncertainty might not have much of an impact on the appropriate management action (*e.g.* catch). This results from the expected biomass having to be close to the target to produce the 50% probability (assuming a moderately symmetrical distribution). However, if the probability statement is that there is a 95% probability that the stock is above a biomass limit, then underestimating uncertainty will allow a higher catch, since the limit is being compared to the tail of the probability distribution.

4. ALTERNATIVE APPROACHES TO IMPLEMENTING THE KOBE STRATEGY MATRIX

There are a number of different analytical approaches that could be used to implement the Kobe strategy matrix. These approaches differ in their computational demands, the types of uncertainty that are represented, and how well they represent the distributional assumptions. Many of the approaches can be modified to take model structural uncertainty into consideration. In general, if the model structural uncertainty can be represented by a model parameter, then it can be automatically taken into consideration by estimating that parameter.

4.1. Monte Carlo methods

The Naïve Monte Carlo method simply takes the model estimated parameters and projects into the future, using random recruitment under different management actions. For each management action, the model is projected many times, using different random numbers, and the probability of exceeding a target is simply the proportion of runs that exceed that target. This approach does not take parameter or model structure uncertainty into consideration, and may therefore greatly underestimate the uncertainty. However, it is likely to require the least computational resources, and it does produce a probability distribution.

Parameter uncertainty can be included in the Monte Carlo method using several approaches, including some of those described below. A simple approach is to randomly select parameters from a multivariate normal distribution parameterized using the variance-covariance matrix, which is available from Stock Synthesis (Methot and Wetzel submitted) due to its use of AD Model Builder (Fournier *et al.* 2012). However, this may not be a very accurate approximation of the true parameter uncertainty, given that the parameterization of Stock Synthesis was not designed for this purpose. Transformation of the parameters may make the multivariate normal a better approximation, but it is currently not feasible for users to modify the Stock Synthesis code. Model structure uncertainty could be implemented by repeating the processes for different model structures and weighting the results by a pre-assigned probability, but this would not take into consideration the support for each model structure provided by the data.

One advantage of the Monte Carlo approach is that, as soon as you have a set of model parameters, which could be different for each stochastic projection, only the projections have to be carried out, and can therefore be repeated for different management strategies without repeating the parameter estimation. The model does not have to re-estimate the model parameters for each projection. The “Puntalizer”, which conducts Monte Carlo forward projections, can be applied to output from Stock Synthesis.

4.2. Bootstrapping

Bootstrapping is a standard technique used to estimate confidence intervals for model parameters or other quantities of interest. The most common approach for stock assessment models is the parametric bootstrap. Essentially, the parametric bootstrap takes the model and the best estimates of the parameters along with the sampling distribution assumptions used in the data-fitting procedure (*e.g.* the likelihood functions) to randomly generate artificial data similar to the observed data. The model is then fitted to these data to estimate the model parameters. The bootstrap is repeated many times and the distribution of the parameter estimates can be used to represent the uncertainty in the parameters and quantities of interest. A Monte Carlo forward projection can be made for each bootstrap to evaluate the different management actions. These projections take into consideration both the parameter uncertainty and the future process variation. The bootstrap does not strictly create a probability distribution, as required in the Kobe strategy matrix, but it can be used as an approximation of a probability distribution. In addition, it can require substantial computer resources, because the model has to be fitted to each artificial data set. Each bootstrap run needs to converge, and discarding unconverged runs can bias results. Stock Synthesis has bootstrap functionality, but not the capability to do Monte Carlo projections, so a secondary program like the “Puntalizer” would have to be used. Once the bootstraps have been conducted, the results can be used to evaluate multiple management strategies without repeating them.

Model structure uncertainty can be taken into consideration by repeating the bootstrap, including the initial parameter estimation, for different model structures, and weighting the results by a pre-assigned probability. This would not take into consideration the support for each model structure provided by the data.

4.3. Bayesian MCMC analysis

Bayesian analysis takes probability statements about model parameters (priors) and updates them with information contained in the data to create posterior probability distributions for parameters and quantities of interest (Punt and Hilborn 1997). These probability statements are ideal for the calculations required by the Kobe strategy matrix. However, Bayesian analysis requires priors for all estimated model parameters, and care needs to be taken that the priors do not have more influence on the results than desired (*e.g.* default priors on parameters for which there is no prior information should not determine the results). The Bayesian analysis can require substantial computational resources for the types of integrated analyses used for tuna stock assessments, due to the large number of parameters and large data sets. Stock Synthesis is based on AD Model Builder, and so automatically has Bayesian inference capabilities. A limited number of prior distribution functional forms are available for all estimated parameters. Forward projections are implemented by treating the projection period as part of the estimation period, but since MCMC produces a set of random draws from the posterior distribution and there are no data in the future, this is equivalent to forward projections using the Monte Carlo method.

The Bayesian approach is the only approach that correctly deals with the historic process variability (random effects models for integrated analyses used in tuna stock assessments are too computationally intensive in non-Bayesian likelihood inference frameworks). Model structure uncertainty can be implemented using reverse jump MCMC, but this is not implemented in Stock Synthesis. Model structure uncertainty can be taken into consideration by repeating the MCMC analysis for different model structures and weighting the results by a pre-assigned probability. However, this would not take into consideration the support for each model structure provided by the data.

4.4. Normal approximation

The uncertainty in estimated parameters and quantities of interest can be approximated by using a normal distribution and an estimate of the standard deviation. Forward projections can be implemented by treating the projection period as part of the historical estimation period (Maunder *et al.* 2006). The future recruitment variation is encapsulated in additional recruitment parameters that are estimated. This allows the inclusion of both parameter uncertainty and process variation. However, there is an inherent bias that needs to be corrected (Maunder *et al.* 2006; Methot and Taylor 2011). In addition, the normal approximation projection method does not adjust for the interaction of the stochastic nature of future recruitment and the stock-recruitment relationship. In a Monte Carlo simulation, a low randomly-selected recruitment leads to low biomass that consequently leads to low expected recruitment based on the stock-recruitment relationship. The normal approximation method only puts confidence intervals around the expected value, and therefore does not allow for this interaction. A major deficiency with the normal approximation method is that the probability distribution is symmetric, which may not be an accurate approximation. Like the bootstrap, the normal approximation method estimates confidence intervals, and therefore only approximates a probability distribution. The method has to be repeated for each harvest strategy, which includes parameter estimation. The estimation can be initiated using the originally estimated parameters to eliminate the estimation period; however, the Hessian matrix still needs to be calculated, which can be very time-consuming.

5. EXAMPLE WITH BIGEYE

We develop an example Kobe strategy matrix for the bigeye tuna stock in the eastern Pacific Ocean (EPO). The stock is assessed using Stock Synthesis (Methot and Wetzel submitted), which has the facility to estimate parameter uncertainty using Bayesian MCMC analysis, bootstraps, and normal approximation.

The stock assessment is conditioned on three model components that are uncertain: natural mortality, length of the oldest fish, and steepness of the Beverton-Holt stock-recruitment relationship. If these quantities are estimated in the model, they are either imprecise or estimated with moderate precision at unrealistic values. Therefore, informative prior information is needed to address these components of the model. Simulation analysis has shown that there is inherent bias in the estimates of steepness in stock assessment models (Con *et al*. 2010; Lee *et al.* 2012). Therefore, steepness should be treated differently so that information from the data does not influence the value of the steepness. Unrealistic results are also encountered when estimating the average length of the oldest bigeye, raising questions about the appropriateness of weighting the values of this parameter by the available data. The structure used for modeling natural mortality cannot be constructed in terms of estimable parameters within Stock Synthesis, forcing the use of model selection sensitivity analysis.

Initial runs using Bayesian analysis took about 10 days to converge. The model was conditioned on fixed values for natural mortality, length of the oldest fish, and steepness. To incorporate the uncertainty about these parameters, the model would have to be either repeated for a range of combinations of values for these parameters, or the parameters also estimated (except steepness). Therefore, it would be computationally infeasible to produce timely Bayesian estimates. The MCMC chain could be split among processes, but each sub-chain would still need a burn-in period, so it is not clear how much reduction in time would be achieved or how many processors would be needed.

The stock assessment model takes about 3.5 hours to converge. Reasonable bootstrap estimates would require several hundred bootstraps. However, due to the issues mentioned previously, the bootstraps would have to be repeated for a combination of natural mortality, average length of the oldest fish, and steepness of the stock-recruitment relationship. Therefore, it would be computationally infeasible to produce timely bootstrap estimates unless a very large number of processes were accessible. Each bootstrap run needs to converge, and discarding unconverged runs can bias results. Given that the models can be unstable, convergence issues might pose a substantial problem.

Due to the computational requirements of the Bayesian and bootstrap methods, we apply the normal approximation method to bigeye tuna in the EPO. We determine the fishing mortality rates relative to the current fishing mortality (*Fscale*) that provide an 80%, 90%, and 95% probability that the spawning biomass (S_y) is above the spawning biomass corresponding to MSY (S_{MSY}) in 5, 10, and 15 years. We also determine the fishing mortality rate that provides an 80%, 90%, and 95% probability that the fishing mortality (F_y) is below the fishing mortality corresponding to MSY (F_{MSY}). Since fishing mortality is the management measure, and we do not model implementation error, the results for fishing mortality are independent of the projection year. We use these two quantities as limit reference points, and define a high probability of not exceeding them, because that is what is implied by the Antigua Convention, which commits the IATTC to applying the precautionary approach, in accordance with the United Nations Fish Stocks Agreement (UNFSA) (Maunder 2012).

To implement the spawning biomass calculations we compare S_y/S_{MSY} with 1. Since Stock Synthesis does not directly calculate S_y/S_{MSY}, the standard deviation of S_y/S_{MSY} has to be calculated from the standard deviations of S_y and S_{MSY}. The variance of the ratio of correlated random variables is approximated by:

$$Var\left(\frac{Y}{X}\right) \approx \left(\frac{\mu_Y^2}{\mu_X^4}\right)\sigma_X^2 + \left(\frac{1}{\mu_X^2}\right)\sigma_Y^2 - 2\left(\frac{\mu_Y}{\mu_X^3}\right)\rho\sigma_X\sigma_Y$$

The spawning biomass calculations are repeated for a range of projected fishing mortality levels. For each fishing mortality, the probability of $S_y/S_{MSY} > 1$ is calculated. A generalized logistic curve is fitted to the relationship of this probability against fishing mortality, and is used to calculate the fishing mortalities corresponding to the desired probabilities.

To implement the fishing mortality calculations, we compare F/F_{MSY} with 1. Since Stock Synthesis does not directly calculate F/F_{MSY}, the standard deviation of F/F_{MSY} has to be calculated from the standard deviation of its inverse F_{MSY}/F. Therefore, we have to calculate the standard deviation of $1/X$, where $X = F_{MSY}/F$. The variance is therefore:

$$Var\left(\frac{1}{X}\right) \approx \left(\frac{1}{\mu_X^4}\right)\sigma_X^2$$

To evaluate the different fishing mortality levels we simply compare $\delta F/F_{MSY}$ with 1, which can be calculated from a single stock assessment run (*i.e.* the stock assessment only has to be run once, and projections do not have to be conducted). Since $Var(aX) = a^2 Var(X)$:

$$Var\left(\frac{\delta F}{F_{MSY}}\right) \approx \delta^2 \left(\frac{1}{\left(F_{MSY}/F\right)^4}\right)\sigma^2_{F_{MSY}/F}$$

Unfortunately, the variance estimate for F_{MSY}/F calculated by Stock Synthesis appears to be unrealistically small. It is not clear whether this is due to the rescaling of the current fishing mortality to sum to one across gears before calculating F_{MSY} or to some other factor. Therefore, we approximate the variance based on the estimate of variance in the exploitation rate, which is available from Stock Synthesis averaged over the most recent three years.

Model structure uncertainty is investigated by applying the fishing mortality calculations to combinations of steepness values of the stock-recruitment relationship, natural mortality, and average length of the oldest bigeye tuna. We apply individual weights to each assumption, and apply the product of these weights to the combinations. These weights are then normalized to sum to one. The distributions for $\delta F/F_{MSY}$ from all the runs are then combined by using these weights (see Table 1) to produce a weighted sum of the cumulative normal distributions:

$$p(\delta F / F_{MSY} < 1) = \sum_i p_i \Phi(x = 1, \mu_{\delta F/F_{MSY,i}}, \sigma^2_{\delta F/F_{MSY,i}})$$

To simplify the calculations we use the standard deviation of F_{MSY}/F from the base case analysis for all the sensitivity runs.

6. RESULTS

To ensure that there is a 95% probability that the spawning biomass in five years is greater than the spawning biomass corresponding to MSY, the fishing mortality has to be reduced by 15% under the base case model (Table 2, Figure 1). The reduction in fishing mortality is less if the time frame is longer or if the desired probability is less. The projections converge to a stable state quickly, so the results are similar for 10 and 15 years. To ensure that there is a 95% probability that the fishing mortality is less than the fishing mortality corresponding to MSY (F_{MSY}), the fishing mortality has to be reduced by 17% (Table 3 and Figure 2). The management quantities are highly sensitive to the model structure uncertainty for natural mortality, average length of the oldest bigeye tuna, and steepness of the stock-recruitment relationship (Figure 3). This model uncertainty greatly reduces the fishing mortality that produces the desired probability of being below F_{MSY} (Table 3, Figure 4). Assigning equal weight to all sensitivities gives more emphasis to extreme assumptions, increasing the uncertainty and further reducing the fishing mortality that produces the desired probability of being below F_{MSY} (Table 3, Figure 5). The management actions are more sensitive to the model structure uncertainty than to choosing between the probabilities of exceeding the reference points.

7. CONCLUSION

The construction of the Kobe strategy matrix for parameter- and data-rich models, such as those used for assessing tunas in the EPO, is computationally intensive, particularly if model structure uncertainty is taken into consideration. The use of the normal approximation method is a practical alternative, as we have shown, but the accuracy of the approximation is unknown. Our results clearly show that ignoring model structure uncertainty, or naïvely including all model structures without appropriately weighting them, can substantially bias the management actions presented in a Kobe strategy matrix. The management actions are more sensitive to the model structure uncertainty than to choosing between the probabilities of exceeding the reference points.

The definition of the Kobe strategy matrix states that "the matrix would present the specific management measures that would achieve the intended management target. The probabilities and timeframes to be evaluated would be determined by the Commission." This implies that the population requires rebuilding in a given amount of time, particularly since it is unlikely that a higher fishing mortality would be implemented and then reduced when the target is met. If this is correct, then perhaps the Kobe strategy matrix is not the appropriate tool for managing stocks that are considered to be healthy.

The Kobe strategy matrix presents only a limited range of probabilities of exceeding a reference point. The risk curves that we have constructed present the whole range of probabilities, and are therefore more informative than the Kobe strategy matrix. Risk curves from different model assumptions can be plotted on the same figure, making the comparisons easier.

Calculating risk curves for fishing mortality rate probabilities when fishing mortality (*i.e*. effort) is the management action is relatively easy, and only moderately computationally intensive when using the normal approximation method, even when accounting for model structure uncertainty. This approach is applicable to bigeye tuna in the EPO because management, at least for the purse-seine fleet, is based on effort control (seasonal fishery closures). However, implementation will require the definition of the desired probability level p($F < F_{MSY}$), the model structures to include, and the probabilities associated with each model structure.

Several improvements are needed for the application of the normal approximation method to bigeye tuna in the EPO. First, the standard deviation for F_{MSY}/F from Stock Synthesis needs to be evaluated to check that it is correct, although this may be simply a misinterpretation of the output on our part. Preferably, the standard deviation for F/F_{MSY} should be calculated within Stock Synthesis. The model structure uncertainty analyses should be repeated using the standard deviations from each run rather than that for the base case. A major deficiency with the normal approximation method is that the probability distribution is symmetric, which may not be an accurate approximation. The other methods do not have this limitation. Simulation tests should be applied to determine the accuracy of the method and whether a transformation would improve the performance. Finally, the analyses presented here are only an illustration of the construction of the Kobe strategy matrix, and the results should not be used for management advice.

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TABLE 1. Relative weights for the different model structure uncertainties for natural mortality, average length of the oldest bigeye tuna, and steepness of the Beverton-Holt stock-recruitment relationship.

Annual natural mortality	Probability	Average length (cm)	Probability	Steepness	Probability
0.3	0.1	175	0.1	0.8	1
0.4	1	180	0.5	0.9	1
0.5	0.1	185	1	1	1
		190	0.5		
		195	0.1		

TABLE 2. Kobe strategy matrix for bigeye tuna, using the base case assessment model, which is conditioned on fixed values of natural mortality, average length of the oldest bigeye tuna, and steepness of the Beverton-Holt stock-recruitment relationship. The contents of the table are the fraction of the current fishing mortality that is required to ensure the given probability that the spawning biomass after a given numbers of years is above the spawning biomass corresponding to MSY.

Management target	Time frame (years)	Probability of meeting target		
		95%	90%	80%
$S>S_{MSY}$	5	0.85	0.92	1.00
	10	0.94	0.99	1.04
	15	0.94	0.99	1.04

TABLE 3. Kobe strategy matrix for bigeye tuna, using the base case assessment and integrating over model structure uncertainty, including values of natural mortality, average length of the oldest bigeye tuna, and steepness of the Beverton-Holt stock-recruitment relationship. The contents of the table are the fraction of the current fishing mortality that is required to ensure the given probability that the fishing mortality is below the fishing mortality corresponding to MSY.

Management target	Weights	Probability of meeting target		
		95%	90%	80%
$F>F_{MSY}$	Base case	0.83	0.86	0.90
	A priori	0.54	0.60	0.67
	Equal	0.43	0.49	0.59

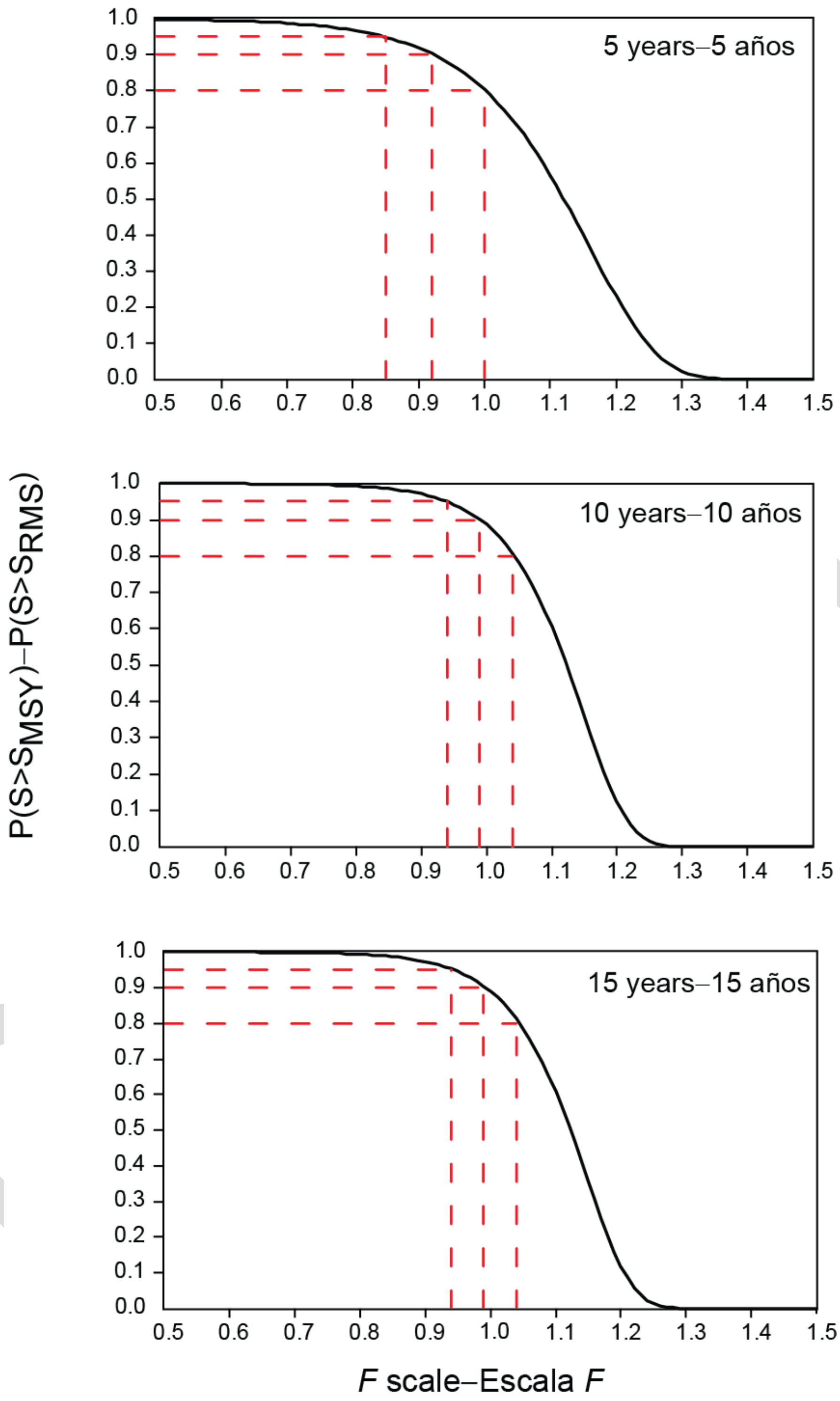

FIGURE 1. Probability that the spawning biomass (*S*) after a given numbers of years is above the spawning biomass corresponding to MSY (S_{MSY}) for different fractions of the current (2009-2011) fishing mortality rate. The dashed lines represent 80%, 90%, and 95% probabilities.

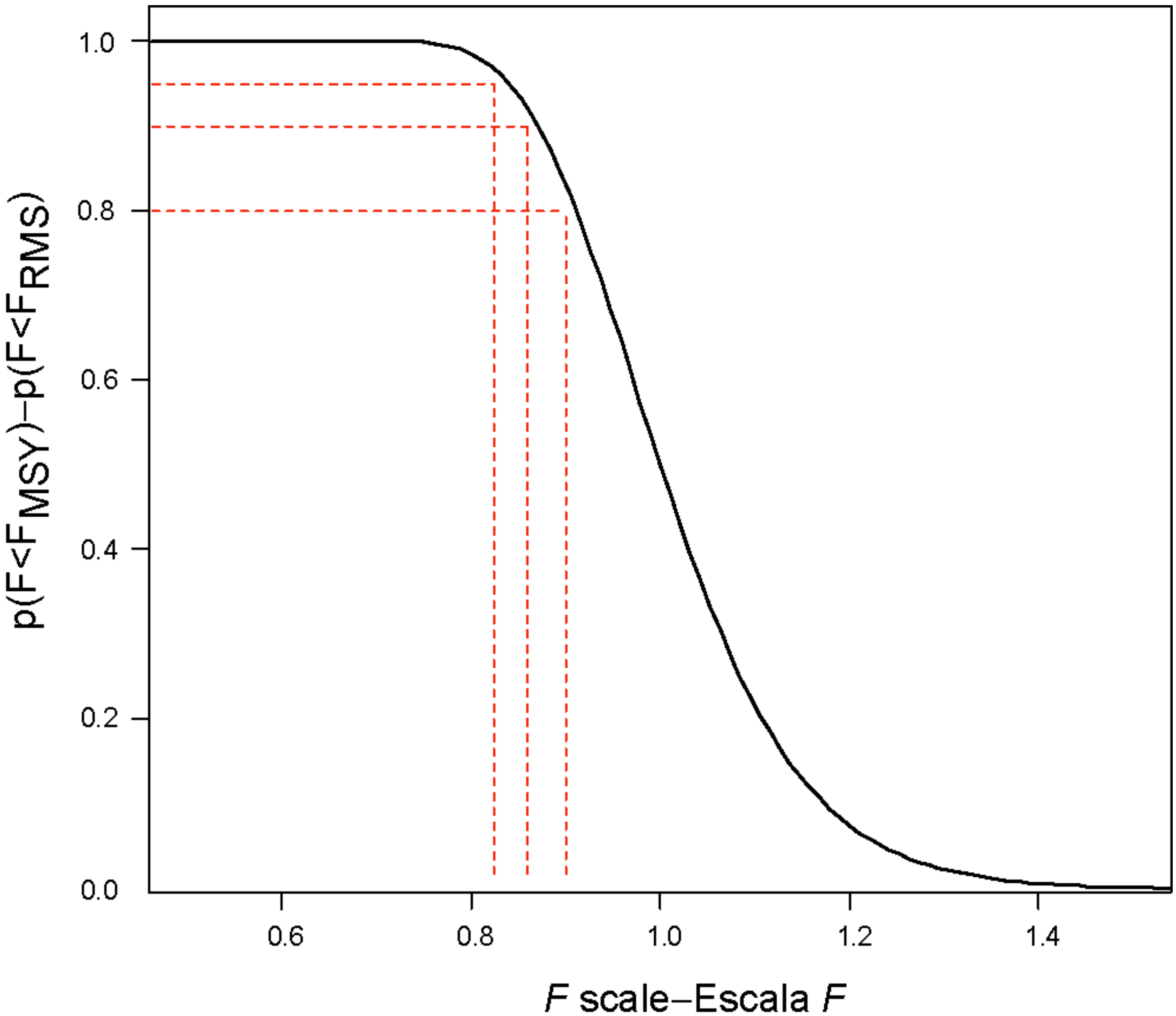

FIGURE 2. Probability that the fishing mortality (*F*) is below the level corresponding to MSY (F_{MSY}) for different fractions of the current (2009-2011) fishing mortality. The dashed lines represent 80%, 90%, and 95% probabilities.

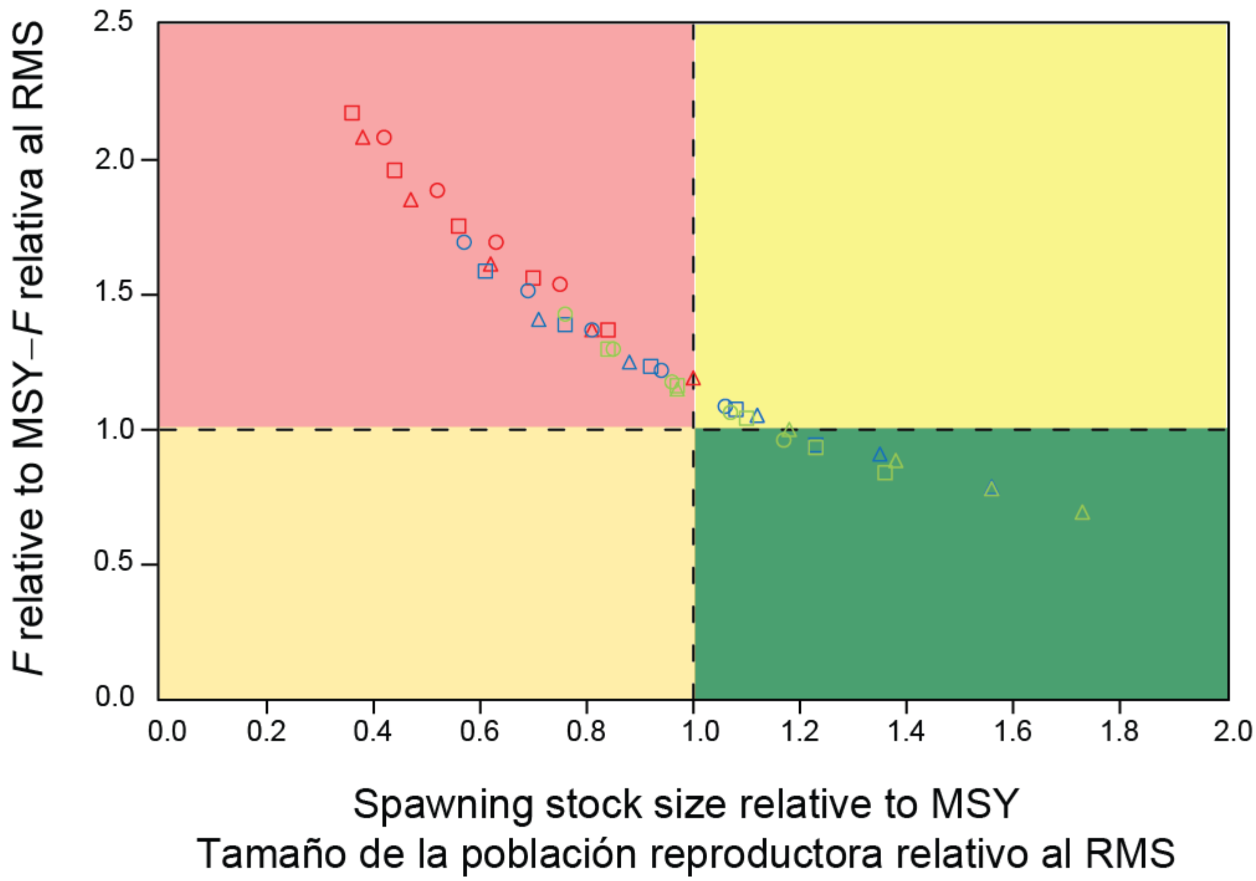

FIGURE 3. Kobe plot of the different sensitivities, with low (red), medium (blue), and high (green) rates of natural mortality (*M*), and with the steepness of the Beverton-Holt stock-recruitment relationship (*h*) set at 0.8 (circles), 0.9 (squares), and 0.1 (triangles); Within a color and shape combination, the points within these categories have different average lengths for the oldest bigeye tuna.

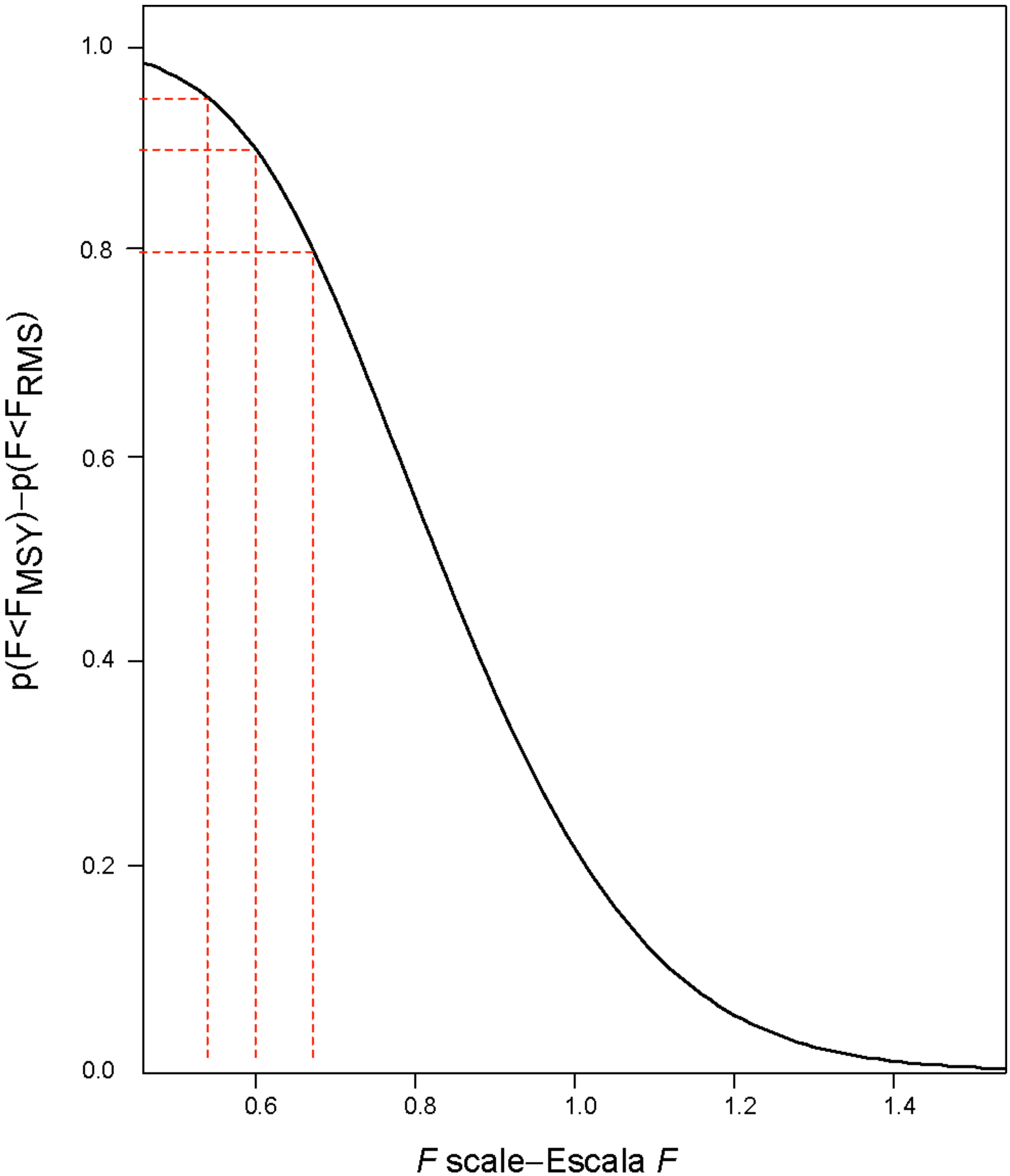

FIGURE 4. Probability that the fishing mortality (*F*) is below the fishing mortality corresponding to MSY (F_{MSY}) for different fractions of the current (2009-2011) fishing mortality rate, integrating over model structure uncertainty. The dashed lines represent 80%, 90%, and 95% probabilities.

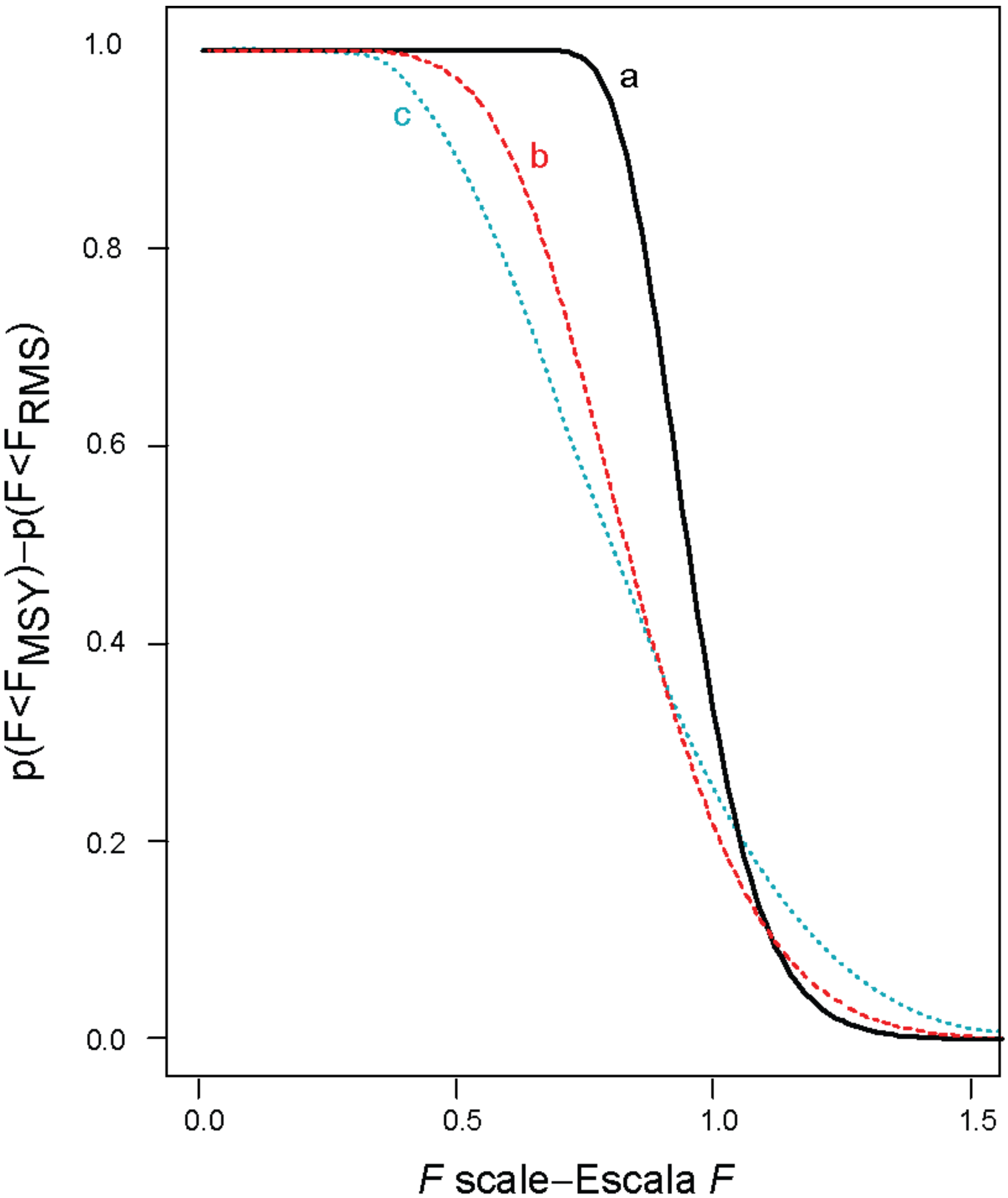

FIGURE 5. Probability that the fishing mortality rate (*F*) is below the fishing mortality corresponding to MSY (F_{MSY}) for different fractions of the current (2009-2011) fishing mortality rate for: (a) the base case assessment (solid line); (b) integrating over model structure uncertainty with *a priori* weights (dashed line); and (c) integrating over model structure uncertainty with equal weighting (dotted line).